Weakly Coupled Grain Model for High T_c Superconducting Thin Films Taking Account of Anisotropic Complex Conductivities

K.Yoshida[1], H.Takeyoshi[1], H.Morita[1], H.Kanaya[1], T.Uchiyama[2], H.Shimakage[2] and Z.Wang[2]

[1]Department of Electronic Device Engineering, Graduate School of Information Science and Electrical Engineering, Kyushu University, 6-10-1 Hakozaki, Higashi-ku, Fukuoka 812-8581, JAPAN

[2]KARC Communications Research Laboratory, 588-2 Iwaoka, Iwaoka-cho, Nishi-ku, Kobe 651-2401, JAPAN

Abstract: An analytical solution of the London equation for the weakly coupled grain model of high-T_c superconducting thin films has been obtained in the case of finite thickness by taking full account of anisotropic conductivities. Using the solution, we provide general expressions for the transmission-line parameters of high-T_c superconducting transmission lines. Dependences of the resistance on the grain size, coupling strength and film thickness have been numerically evaluated and discussed.

1. Introduction

Intensive studies have been made of applications of high-T_c superconducting transmission lines, which have low loss and low dispersion characteristics, to microwave devices for mobile and satellite communications. In these applications the precise evaluation of the transmission-line parameters of superconducting thin films such as the resistance and the kinetic inductance, which result from the complex conductivity, is essential. It has been pointed out by Hylton et al (1988)(1989a)(1989b) that these basic parameters are greatly influenced by the weaklinks(Josephson junctions) inevitably existing in high-T_c thin films, i.e., the so-called "weakly coupled grain (WCG) model". As a result of this model, it is supposed that the weaklink gives rise to the excess surface resistance (Attanasio et al 1991)(Miller et al 1988)(Nguyen et al 1993)(Oates et al 1993) and tends to enhance the kinetic inductance of polycrystalline films compared with that of the single crystal (Porch et al 1993)(Yoshida et al (1996)(1998)). Detailed studies of this model, however, have not been made, especially in terms of anisotropic properties of high T_c superconducting films.

In our previous papers (Yoshida et al (1996)(1998)) we have carried out experimental studies on the WCG model neglecting the anisotropy of high T_c superconducting films. In this paper we make, for the first time, a general theoretical formulation of the effects of anisotropy on the WCG model. In Sec.2 an analytical solution of the London equation for the weakly coupled grain model of high T_c superconducting thin films has been obtained in the case of finite film thickness by taking full account of anisotropic conductivities. Using the solution, we obtained the expressions for the transmission-line parameters of high T_c superconducting transmission lines in Sec.3. Dependences of the resistance on the grain size, coupling strength and film thickness have been numerically evaluated and discussed.

2. Expression for the Magnetic Field Distribution in the Anisotropic Superconducting Thin Film with Grain Boundaries

In Fig.1 we show the schematic figure of the weakly coupled grain (WCG) model proposed by Hylton et al (1988) for a c-axis oriented high T_c superconducting film, where the c-axis of the superconducting film is perpendicular to the substrate. The average grain size is assumed to be a, and the film thickness is d.

In Fig.2 we show the cross section of the superconducting film representing the WCG model, where the external current K [A/m] per unit length in the y direction is assumed to be flowing in the z direction.

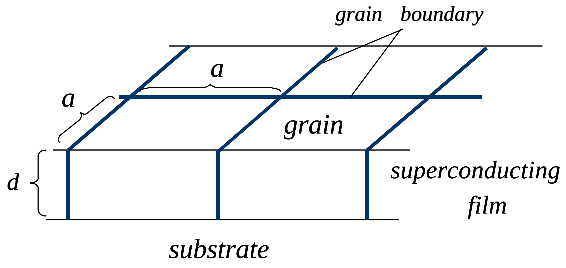


Fig. 1 Schematic of weakly coupled grain (WCG) model.

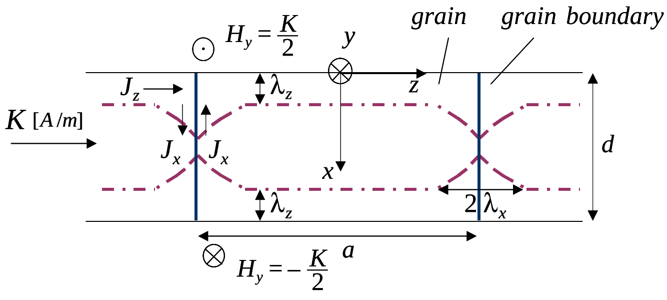


Fig.2 The cross section of the superconducting thin film

The London equation for the anisotropic superconductor is given as (Hylton et al 1989)(Van Duzer et al 1981)

$$\lambda_x^2 \frac{\partial^2 H_y}{\partial z^2} + \lambda_z^2 \frac{\partial^2 H_y}{\partial x^2} = H_y \qquad (1)$$

with the boundary condition at the grain boundary :

$$\frac{\partial^2 H_y}{\partial x^2} \mp \frac{\lambda_x}{\lambda_J^{(0)2}} \frac{\partial H_y}{\partial z} = 0 \quad at \quad z = \pm \frac{a}{2} \qquad (2)$$

with

$$\lambda_J^{(0)} = \sqrt{\frac{\Phi_0}{2\pi \mu_0 2\lambda_x J_c}}$$

where λ_x and λ_z are the magnetic penetration depth in the x and z direction, respectively, J_c is the critical current density for the Josephson junction and $\lambda_J^{(0)}$ corresponds to the penetration depth for an isolated single Josephson junction. According to the symmetry and periodicity of the present problem, other boundary conditions for a single grain located in the region $-a/2 \leqq z \leqq a/2$ can be expressed as

$$H_y(x=0) = -\frac{K}{2} \qquad (3)$$

$$H_y(x=d) = \frac{K}{2} \qquad (4)$$

The solution of Eq.(1) satisfying boundary conditions Eqs.(2)-(4) is obtained as :

$$\frac{H_y(x,z)}{(K/2)} = \frac{\sinh \frac{1}{\lambda_z}(x - \frac{d}{2})}{\sinh (\frac{d}{2\lambda_z})} -$$

$$\sum_{n=1}^{\infty} \frac{8n\pi}{d^2} \frac{1}{[1 + \lambda_z^2(\frac{2n\pi}{d})^2]} \frac{\sin(\frac{2n\pi}{d}x) \cosh (\frac{\sqrt{1 + \lambda_z^2(\frac{2n\pi}{d})^2}}{\lambda_x} z)}{[(\frac{2n\pi}{d})^2 \cosh (\frac{a}{2\lambda_x}\sqrt{1 + \lambda_z^2(\frac{2n\pi}{d})^2}) + \frac{\sqrt{1 + \lambda_z^2(\frac{2n\pi}{d})^2}}{\lambda_J^2} \sinh (\frac{a}{2\lambda_x}\sqrt{1 + \lambda_z^2(\frac{2n\pi}{d})^2})]} \quad (5)$$

It is shown that in the limit of $d = +\infty$, Eq.(5) coincides with the solution given by Hylton et al (1989a) valid only for the case of a semi-infinite superconductor. Using this solution, the current distribution can be obtained from the Maxwell equation $\mathbf{J} = \nabla \times \mathbf{H}$.

3. The Equivalent Circuit for the High T_c Superconducting Transmission Line with Grain Boundaries

For the case of an applied alternating current as $Ie^{j\omega t}$, we obtain the expression for the total current density **J** as

$$\boldsymbol{J} = \boldsymbol{J_N} + \boldsymbol{J_S} = ([\sigma_1] - j[\sigma_2])\, \boldsymbol{E} \qquad (6)$$

with

$$[\sigma_1] = \begin{pmatrix} \sigma_{1x} & 0 & 0 \\ 0 & \sigma_{1y} & 0 \\ 0 & 0 & \sigma_{1z} \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho_c} & 0 & 0 \\ 0 & \frac{1}{\rho_{ab}} & 0 \\ 0 & 0 & \frac{1}{\rho_{ab}} \end{pmatrix}$$

$$[\sigma_2] = \begin{pmatrix} \sigma_{2x} & 0 & 0 \\ 0 & \sigma_{2y} & 0 \\ 0 & 0 & \sigma_{2z} \end{pmatrix} = \begin{pmatrix} \frac{1}{\omega\mu_0\lambda_c^2} & 0 & 0 \\ 0 & \frac{1}{\omega\mu_0\lambda_{ab}^2} & 0 \\ 0 & 0 & \frac{1}{\omega\mu_0\lambda_{ab}^2} \end{pmatrix}$$

where $\mathbf{J_N}$ is the normal-conducting current density, $\mathbf{J_S}$ is the superconducting current density, **E** is the electric field, μ_0 is the vacuum permeability, $\lambda_x = \lambda_c$ is the magnetic penetration depth along the c-axis and $\lambda_y = \lambda_z = \lambda_{ab}$ is the magnetic penetration depth in the a-b plane, $[\sigma_1]$ is the normal conductivity tensor, $\sigma_{1x} = 1/\rho_c$, $\sigma_{1y} = \sigma_{1z} = 1/\rho_{ab}$, ρ_c is the resistivity in the c direction and ρ_{ab} is the resistivity in the a-b plane, $[\sigma_2]$ is the superconductivity tensor, ω is the angular frequency.

In Fig.3 we show the equivalent circuit for the transmission line with a unit length made of anisotropic high T_c superconducting films including grain boundaries. In this figure L_m represents the conventional magnetic inductance per unit length, which is almost independent of the conductor material, and L_{kG} and L_{kJ} denote the kinetic inductance of the superconductor per unit length for the grain and the grain boundary, respectively. R_G and R_J represent the resistance per unit length resulted from the grain and the grain boundary, respectively.

The expressions for the kinetic inductance and the resistance of the grain and the junction can be obtained by calculating the kinetic energy of the superconducting electrons and the balance of the power consumption, respectively :

$$\frac{1}{2} L_K |I|^2 = \int \mathrm{Re}\,[\tfrac{1}{2}\mu_0[\lambda^2]\, \boldsymbol{J}_S \cdot \boldsymbol{J}_S{}^*]\, dv \qquad (7)$$

$$\frac{1}{2} R |I|^2 = \int \mathrm{Re}\,[\tfrac{1}{2}(\boldsymbol{J}\cdot\boldsymbol{E}^*)] dv \qquad (8)$$

where I is the total current as shown in Fig.3 and the volume integral extends over unit length in the z direction. The obtained results are :

$$R_G = \frac{1}{D}\left(\frac{\omega L_{kx}}{Q_x\left(1+Q_x^{-2}\right)} + \frac{\omega L_{kz}}{Q_z\left(1+Q_z^{-2}\right)} \right) \qquad (9)$$

$$R_J = \frac{1}{D}\left(\frac{\omega L_{kJ}}{Q_J\left(1+Q_J^{-2}\right)} \right) \qquad (10)$$

$$L_{kG} = \frac{1}{D}\left(\frac{L_{kx}}{1+Q_x^{-2}} + \frac{L_{kz}}{1+Q_z^{-2}} \right) \qquad (11)$$

$$L_{kJ} = \frac{1}{D}\frac{L_{kJ}^{(0)}}{\left(1+Q_J^{-2}\right)} \qquad (12)$$

with

$$L_{kx} = \int_0^d dx \frac{1}{a}\int_{-\frac{a}{2}}^{\frac{a}{2}} dz\; \mu_0 \lambda_x^2 f_x^2(x,z)$$

$$L_{kz} = \int_0^d dx \frac{1}{a}\int_{-\frac{a}{2}}^{\frac{a}{2}} dz\; \mu_0 \lambda_x^2 f_z^2(x,z)$$

$$Q_x = \frac{\sigma_{2x}}{\sigma_1}$$

$$Q_z = \frac{\sigma_{2z}}{\sigma_{1z}}$$

$$Q_J = \frac{R_N}{\omega L^{(0)}} = \frac{I_c R_N}{f\,\Phi_0}$$

$$L_J^{(0)} = \frac{\Phi_0}{2\pi I_c}$$

$$L_{kJ}^{(0)} = \int_0^d dx\; \mu_0 \lambda_J^2 \left| J_z(x,\tfrac{a}{2})\right|^2$$

$$\lambda_J = \sqrt{\frac{\Phi_0}{2\pi\mu_0 a J_c}}$$

where Q_x and Q_z represent the quality factor for the current component J_x and J_z in the grain, Q_J represents the quality factor of the junction, R_N is the junction resistance, $L_J^{(0)}$ is the kinetic inductance for a single junction, I_c is the junction critical current, L_{kx} and L_{kz} are the kinetic inductance per unit area (sheet inductance) associated with the current component J_x and J_z, respectively, and λ_J corresponds to the Josephson penetration depth in the small grain limit (Hylton et al 1989a). The quantities f_x and f_z represent the current density for the case of a unit applied current, i.e, K=1 : They are defined by

$$J_x(x,y,z) = f_x(x,z)\,K(y) \tag{13}$$

$$J_z(x,y,z) = f_z(x,z)\,K(y) \tag{14}$$

Equations (9), (10) indicate that the resistances are proportional to the respective kinetic inductances.

The quantity D in Eqs.(9)-(12) is defined by

$$D = \frac{|I|^2}{\int_{-\infty}^{\infty} K^2(y)dy} \tag{15}$$

which corresponds to the geometrical factor representing the characteristic length for a particular transmission line geometry. The characteristic length D defined by Eq.(15) depends on the geometry of the transmission line. If we assume that the film thickness is sufficiently thin and that the characteristic impedance is 50Ω on MgO substrate with the permitivity $\varepsilon_r = 9.4$, D is determined by the geometrical configuration of the transmission line almost independent of the internal structure of the conductor, and we can obtain numerically following the procedure given in Sheen et al (1991) :

D = 0.4 W for coplanar waveguide
D = 0.5 W for micro-stripline

where W is the width of the signal electrode.

In order to evaluate the values for the resistance for various parameters, we first introduce the following normalized parameters ; $\alpha = a/(2\lambda_x)$, $\beta = \lambda_z^2/\lambda_J^{(0)2}$, $\gamma = d/(2\lambda_z)$, where α is the normalized grain size, $\beta = \lambda_z^2/\lambda_J^{(0)2} = (4\pi\mu_0/\Phi_0)\lambda_x\lambda_z^2 J_c$ represents the strength of the coupling and γ is the normalized thickness.

In Figs.4,5 and 6 we show the dependence of the resistances per unit length on α, β and γ, respectively. In the calculation we used the typical experimental values for $YBa_2Cu_3O_x$ superconductors (Friedmann et al 1990)(Hylton et al 1989) : $\lambda_{ab} = 0.15$ [μm], $\lambda_c = 0.5$ [μm], $\rho_c = 2.5\times10^{-5}$ [Ωm], $\rho_{ab} = 4.0\times10^{-7}$ [Ωm], which leads to $Q_x = 1.3\times10^4$ and $Q_z = 2.3\times10^3$ at $\omega = 2\pi\times10^9$ [Hz]. Junction parameters are : $Q_J = 4.8\times10^2$ for it I_cR_N=1 [mV] and f =10^9 [Hz]. It must be mentioned that Eqs.(9) and (10) lead to the resistances proportional to ω^2 in the case of Q_x , Q_z , Q_J >> 1. The resistances shown in Figs.4,5 and 6 are calculated at $\omega = 2\pi\times10^9$ [Hz] in the case of D=100 [μm].

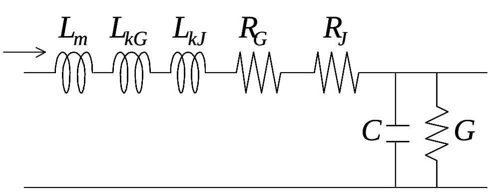


Fig.3 The equivalent circuit for the transmission line with a unit length

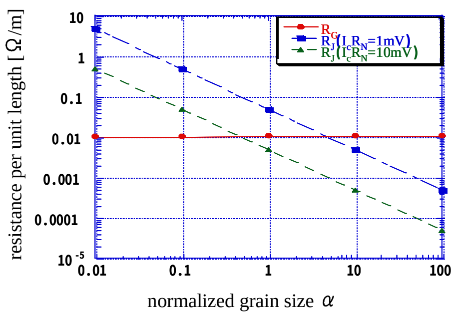
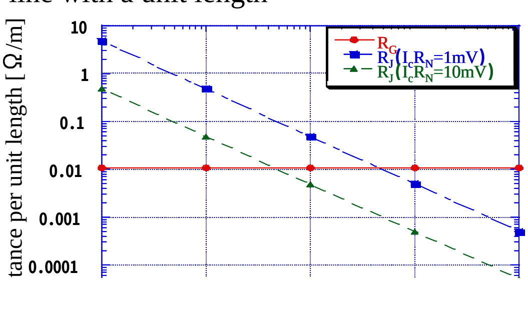


Fig.4 The dependence of the resistance per unit length on α for β = γ =1.



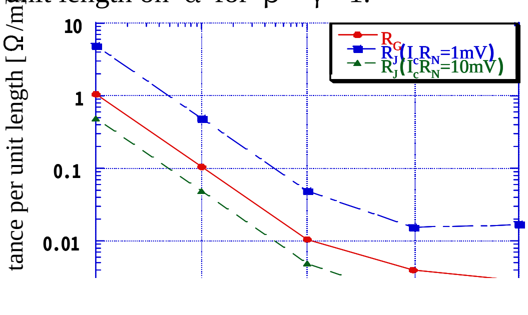


Fig.5 The dependence of the resistance per unit length on β for $\alpha = \gamma = 1$.

Fig.6 The dependence of the resistance per unit length on γ for $\alpha = \beta = 1$.

4. Conclusions

In the present paper we obtained general expressions for the the resistance and the kinetic inductance of the superconducting thin films with grain boundaries by taking full account of anisotropic conductivities, which include the normalized grain size α, coupling strength β and normalized thickness γ as free parameters. Detailed discussions with specific samples are to be our next work.

References

Attanasio C, Maritato L and Vaglio R 1991 Phys. Rev.B, **43**, no.7, pp.6128-6131

Friedmann T A, Rabin M W, Giapintzakis J, Rice J P and Ginsberg D M 1990 Phys. RevB, **42**, no10 , pp.6217-6221

Hylton T L, Kapitulnik A and Beasley M R 1988 Appl.Phys.Lett., **53**, no.14, pp.1343-1345

Hylton T L and Beasley M R 1989a Phys. Rev. B., **39**, no.13, pp.9042-9048

Hylton T L, Beasley M R, Kapitulnik A, Carini J P, Drabeck L and Gruner G 1989b IEEE Trans.Magn., **25**, no.2, pp.810-813

Miller D, Richards P.L, Etad S, Inam A, Venkatesan T, Dutta B, Wu X D, Eom C B, Geballe T H, Newman N and Cole B G 1988 Appl. Phys. Lett., **59**, no.18, pp.2326-2328

Nguyen P P, Oates D E, Dresselhaus G, and Dresselhaus M.S 1993 Phys. Rev. B, **48**, no.9, pp.6400-6412

Oates D E, Nguyen P P, Dresselhaus G, Dresselhaus M S, and Chin C C 1993 IEEE Trans. Appl. Supercond., **3**, no.1, pp.1114-1117

Porch A, Lancaster M J, Humphreys R G and Chew N G 1993 IEEE Trans. Appl. Supercond., **3**,no.1, pp.1719-1722

Sheen D M, Ali S M, Oates D E, Withers R S and Kong J A 1991 IEEE Trans. Appl. Supercond., **1**, No.2, pp.108-115

Van Duzer T and Turner C W 1981 Principles of Superconductive Devices and Circuits, Elsevier North Holland Inc., New York

Yoshida K, Onoue T, Kiss T, Shimakage H and Wang Z 1996 IEICE Trans. Electron., **E79-C**, no.9, pp.1254-1259

Yoshida K, Adou T, Nishioka S, Kanda Y, Shimakege H and Wang Z 1998 IEICE Trans.Electron., **E81-C**, no10 , pp.1565-1572