# Broadband and Low Driving-Voltage $\mathrm{LiNbO}_{3}$ Optical Modulator with High Tc Superconducting Transmission Line 

K.Yoshida, H.Takeuchi, H.Kanaya, Y.Kanda, T.Uchiyama and Z.Wang


#### Abstract

A new design theory is presented of the high- $\boldsymbol{T}_{\boldsymbol{c}}$ superconducting broadband booster circuit for a $\mathrm{LiNbO}_{3}$ optical modulator. Based on this theory we studied the gain and bandwidth characteristics of the modulator using a transmission line model. We also designed the practical booster circuit using an electromagnetic wave simulator and demonstrated the optical modulator with broadband and high voltage gain as expected from the theory.


Index Terms_ broadband and low driving-voltage optical modulator, high- $T_{c}$ superconducting transmission line, $\mathrm{LiNbO}_{3}$ optical modulator, microwave applications of high- $T_{c}$ superconductors.

## I. INTRODUCTION

Extensive studies have been made of applications of high$T_{c}$ superconducting transmission lines with low loss and low dispersion to microwave passive devices such as filters, resonators and antennas. We have studied so far the applications of superconducting transmission lines to traveling-wave-type $\mathrm{LiNbO}_{3}$ (LN) optical modulators with broad bandwidth, and it has been demonstrated that the optical modulator with superconductors is far superior to that using normal-conductors [1-2].

Recently, sub-carrier optical transmission systems in which microwave or millimeter-wave signals are carried through optical fibers by intensity-modulated lightwave (radio on fiber) has been studied intensively [3-4]. Resonant type optical modulators are suitable for this purpose [5], and preliminary experiments using YBCO films has been made[6-7]. It has been revealed, however, that the bandwidth of the resonant type optical modulator is liable to be narrow due to the high $Q$ resonance of the superconducting resonator.

In this paper, we present a new design theory of broadband booster circuit for a LN optical modulator, which is necessary to amplify the signal voltage applied to the LN optical waveguide. Based on this theory we studied the gain and bandwidth of the booster circuit using the circuit model with coplanar waveguide (CPW) transmission lines (transmission

## Manuscript received Sep.18, 2000.

K. Yoshida, H. Takeuchi and H. Kanaya are with Department of Electronics, Graduate School of Information Science and Electrical Engineering, Kyushu University, Fukuoka, 812-8581, Japan ( e-mail:yoshida@ed.kyushu-u.ac.jp). Y. Kanda is Department of Electronic Device Engineering, Fukuoka Institute of Technology, Fukuoka, 811-0295, Japan.
T. Uchiyama and Z. Wang are KARC Communications Research Laboratory, Kobe, 651-2401, Japan.
line model), designed the practical device with the electromagnetic wave (EM) simulator, and demonstrated the performance expected from the theory.

## II. Design of Broadband Booster Circuit for $\mathrm{LiNbO}_{3}$ Optical Modulator

## A. Expression for Modulation Depth for Standing Wave Signal Voltage

In Fig. 1 we show the schematic figure of the LN optical modulator with the broadband booster circuit made of superconducting CPW transmission lines. The signal voltage is applied to the Mach-Zehnder type optical waveguide via the standing-wave voltage occurring in the CPW which is short circuited at the end terminal as shown in Fig. 2(a).

If we take the coordinate system as shown in Fig. 2(b), assuming the shorted end at $x=l$, the expressions for the spatial profile of the complex amplitude of current $I(x)$ and voltage $V(x)$ are given by

$$
\begin{align*}
& I(x)=I(0) \frac{\cosh \gamma(l-x)}{\cosh \gamma l},  \tag{1}\\
& V(x)=Z_{0} I(0) \frac{\sinh \gamma(l-x)}{\cosh \gamma l} \tag{2}
\end{align*}
$$

where $\gamma=\alpha+j \beta$ is the propagation constant, $\alpha$ is the attenuation constant, $\beta=\omega \sqrt{L C}$ is the phase constant, $\omega$ is the angular frequency, $L$ and $C$ are the inductance and capacitance of the CPW transmission line per unit length, respectively, and $Z_{0}=\sqrt{L / C}$ is the characteristic impedance. $]$

In this case the input impedance at $x=0, \quad Z=V(0) / I(0)$, is given by

$$
\begin{equation*}
Z=Z_{0} \tanh \gamma l . \tag{3}
\end{equation*}
$$

In the presence of the standing-wave voltage given by (2), the induced change of the refractive index $\Delta n(\mathrm{x}, \mathrm{t})$ of LN substrate is given by [5]

$$
\begin{equation*}
\Delta n(x, t)=\frac{1}{2} n_{0}^{3} \gamma_{33} \frac{\Gamma}{d} v(x, t) \tag{4}
\end{equation*}
$$

where $n_{0}$ is the refractive index of $\mathrm{LN}, \gamma_{33}$ is the electro-optic coefficient, $\Gamma$ is the overlap integral, $d$ is the spacing between signal and ground electrodes, and $v(x, t)$ is the instantaneous voltage waveform corresponding to (2):

$$
\begin{equation*}
v(x, t)=\frac{1}{2 \pi} \int_{-\infty}^{\infty} V(x) e^{j \omega t} d \omega \tag{5}
\end{equation*}
$$

In this case the expression for the induced total optical phase difference $\Delta \phi(\mathrm{t})$ at the output end of the push-pull type optical modulator can be calculated as [5]

$$
\begin{align*}
\Delta \phi(t) & =\frac{4 \pi}{\lambda} \int_{0}^{l} \Delta n\left(x, \frac{n_{0}}{c} x+t\right) d x \\
& =\frac{\pi}{V_{\pi}} \frac{1}{2 \pi} \int_{-\infty}^{\infty} Z_{0} I(0) M(\omega) e^{j \omega t} d \omega \tag{6}
\end{align*}
$$

where $\lambda$ is the light wavelength, $V_{\pi}=\lambda d / 2 \Gamma n_{0}{ }^{3} \gamma_{33} l$ is the half-wavelength voltage, and $M(\omega)$ is the term representing the magnitude of the coupling between signal and the optical fields given as

$$
\begin{equation*}
M(\omega)=\frac{1}{l} \int_{0}^{l} \frac{\sinh \gamma(l-x)}{\cosh \gamma l} e^{j \frac{n_{0}}{c} \omega x} d x \tag{7}
\end{equation*}
$$



Fig.1. Schematic of the broadband booster circuit for the Mach-Zehnder type $\mathrm{LiNbO}_{3}$ optical modulator
(a)

(b)


Fig.2. (a) Coplanar waveguide short circuited at the end, (b) Outline of the standing-wave voltage $V(x)$ and current $I(x)$ occurring in the coplanar waveguide $(0 \leq x \leq l)$ short circuited at the end terminal $x=l$. $Z=V(0) / I(0)$ is the input impedance at $x=0$.

## B. Design of Broadband Booster Circuit

In order to realize a broadband booster circuit for exciting a large standing-wave voltage in the short circuited CPW as shown in Fig.2(a), we take advantage of the design theory for the filter circuit with zero source impedance [8] as shown in Fig. 3, where $2 V_{i}$ is the signal voltage.

In this case of an n-pole bandpass filter the formulas for the
admittance inverters are given by

$$
\begin{align*}
J_{0,1} & =\sqrt{w} \sqrt{\frac{b_{1}}{Z_{0} g_{1}}} \\
J_{i, i+1} & =w \sqrt{\frac{b_{i} b_{i+1}}{g_{i} g_{i+1}}}, \quad(i=1,2, \cdots, n-1)  \tag{8}\\
J_{n, n+1} & =\sqrt{w} \sqrt{\frac{b_{n}}{Z_{0} g_{n}}}
\end{align*}
$$

where $g_{i}(i=1,2, \cdots, n)$ are normalized element values for prototype lowpass filter with zero source impedance given in [8], $\quad w=\left(\omega_{2}-\omega_{1}\right) / \omega_{0}$ is the normalized band width, $\omega_{1}$ and $\omega_{2}$ are cutoff frequencies, $\omega_{0}=\sqrt{\omega_{1} \omega_{2}}$ is the center frequency, $B_{\mathrm{i}}(i=1,2, \cdots, n)$ are the susceptance of the parallel resonant circuit expressed as

$$
\begin{equation*}
B_{i}=b_{i}\left(\frac{\omega}{\omega_{0}}-\frac{\omega_{0}}{\omega}\right), \tag{9}
\end{equation*}
$$

where $b_{i}=\left.\left(\omega_{0} / 2\right)\left(\partial B_{i} / \partial \omega\right)\right|_{\omega=\omega_{0}}$ is the susceptance slope parameter.
In the theory given by (8), it is noted that the output voltage $V_{\text {out }}$ shown in Fig. 3 is designed as : $\left|V_{\text {out }} / V_{i}\right|=1$ for passband and $\left|V_{\text {out }} / V_{i}\right|=0$ for offband.
We can apply this filter circuit for exciting the standing-wave voltage in the short circuited CPW whose input impedance is $Z$ as shown in Fig. 2. By applying the reciprocity theorem and making use of the admittance inverters, we propose a new type of broadband booster circuit for the load impedance $Z$ as shown in Fig. 4. The design values for the admittance inverters $J_{\mathrm{i}, \mathrm{i}+1}$ are the same as (8) except for the terms with $i=n-1$ and $i=n$ : The new design values are given by,

$$
\begin{align*}
& J_{n-1, n}^{\prime}=w \sqrt{\frac{b_{n-1} b_{n}^{\prime}}{g_{n-1} g_{n}}} \\
& J_{n, n+1}^{\prime}=\sqrt{w} \sqrt{\frac{b_{n}^{\prime}}{Z_{0} g_{n}}} A, \tag{10}
\end{align*}
$$

with

$$
\begin{equation*}
b_{n}^{\prime}=\frac{b_{n}}{1-\frac{w x A^{2}}{Z_{0} g_{n}}} \tag{11}
\end{equation*}
$$

where we assume that in the vicinity of series resonance the input impedance given in (3) can be expressed as

$$
\begin{equation*}
\operatorname{Im}[Z]=x\left(\frac{\omega}{\omega_{0}}-\frac{\omega_{0}}{\omega}\right) \tag{12}
\end{equation*}
$$

where

$$
x=\left.\left(\omega_{0} / 2\right)(\partial \operatorname{Im}[Z] / \partial \omega)\right|_{\omega=\omega_{0}}=(\pi / 2) Z_{0} \quad \text { is the }
$$

reactance slope parameter of the series resonant circuit, $\omega_{0}=\pi / l \sqrt{L C}$ is the resonance frequency, and $A(>1)$ is the parameter determining the voltage gain.

It is noted that in this design values the maximum voltage amplitude in Fig. 2(b), $V_{\max }=Z_{0} I(0)$, is given as: $\left|V_{\text {max }} / V_{i}\right|=A$ for passband and $\left|V_{\max } / V_{i}\right|=0$ for offband. It must be also mentioned that (11) gives the intrinsic theoretical limit of the gain-bandwidth product as $A^{2} w<Z_{0} g_{n} / x$. Within this limit, however, we can design broadband and large voltage-gain booster circuits as shown in Sec.III.

In order to realize the parallel resonance circuits and inverters shown in Fig. 4 we use half-wavelength resonators and gaps in CPW transmission lines [9]. If we use this coupling section, which is the last resonator of the CPW transmission line, for the optical modulator shown in Fig. 1, the induced optical phase difference given by (6) can be rewritten as

$$
\begin{equation*}
\Delta \phi(t)=\frac{\pi}{V_{\pi}} \frac{1}{2 \pi} \int_{-\infty}^{\infty} F(\omega) V_{i}(\omega) e^{j \omega t} d \omega, \tag{13}
\end{equation*}
$$

with

$$
\begin{equation*}
F(\omega)=G(\omega) M(\omega), \tag{14}
\end{equation*}
$$

where $F$ is referred to as the normalized modulation depth, $G(\omega)=Z_{0} I(0) / V_{i}=V_{\text {max }} / V_{i}$ is the transfer function for the voltage gain, where $V_{i}(\omega)$ is the Fourier transform of the incident voltage waveform $v_{i}(t)$, i.e.,

$$
\begin{equation*}
V_{i}(\omega)=\int_{-\infty}^{\infty} v_{i}(t) e^{-j \omega t} d t \tag{15}
\end{equation*}
$$



Fig.3. Prototype $n$-pole bandpass filter with signal voltage $2 V_{\mathrm{i}}$ and zero source impedance.


Fig.4. Proposed $n$-pole broadband booster circuit with load impedance $Z$. The generator has the signal voltage $2 V_{\mathrm{i}}$ and the source impedance $Z_{0}$.

## III. Design and Performance of the Broadband Optical Modulator with Transmission Line Model

present broadband optical modulator expected form the electrical circuit using the transmission line model.

## A. Frequency Dependence of Modulation Depth

In Fig. 5(a) we show the frequency dependence of the magnitude of the modulation depth $|F(\omega)|$ given by (14) for $w=0.1 \%$ and $w=1 \%$ in the case of Chebyshev filter with $n=3$, $\omega_{0} / 2 \pi=10 \mathrm{GHz}$ and the ripple of the insertion loss $\mathrm{Ar}=0.01 \mathrm{~dB}$. In Fig. 5(b) we also show the frequency dependence of $|F(\omega)|$ with different voltage gains of $A=1$ and $A=5$. It is shown that the bandwidth and the voltage gain can be adjusted independently.
(a)

$\begin{array}{lllllll}9.7 & 9.8 & 9.9 & 10 & 10.1 & 10.2 & 10.3\end{array}$
Frequency[GHz]
(b)


Fig. 5 (a) Frequency dependence of the modulation depth $|F(\omega)|$ for $w=0.1 \%$ and $w=1 \%$, (b) Frequency dependence of the modulation depth $|F(\omega)|$ for $A=1$ and $\mathrm{A}=5$.

## B. Time Domain Characteristics of the Optical Modulator

We can calculate the temporal waveforms of the output optical phase difference $\Delta \phi(t)$ against the input signal voltage $v_{i}(t)$ using (7), (13), (14) and (15).

In Fig. 6(a) we show the example of the input voltage waveform $v_{i}(t)$ with ASK (Amplitude Shift Keying) modulation signal, and calculated output phase difference $\Delta \phi(t)$ for the cases of $A=1$ and $A=5$ are shown in Fig. 6(b) and Fig. 6(c), respectively. It is shown that designed voltage gains are confirmed, indicating that a low driving-voltage optical modulator is possible by designing a large voltage gain.

In this section we show the theoretical performance of the
(a)

(c)

Fig. 6 Time domain characteristics of the optical modulator, (a) input voltage waveform $v_{i}(t)$ of ASK signal, (b) output optical phase difference in the case of $A=1$, (c) output optical phase difference in the case of $A=5$

## IV. Simulated Performance of the Practical Optical MODULATOR

In this section we show the design and performance of the practical optical modulator using the EM simulator (HP Momentum).

In Fig. 7(a), we show the configuration of the booster circuit made of meander CPW transmission lines. The length of the half-wavelength resonator and the geometry of inverters are decided in the same manner as described in [9].

In Fig. 7(a) and 7(b) we show the spatial distribution of the amplitude of the standing wave voltage calculated by the EM simulator and the transmission line model, respectively. A reasonable agreement is obtained for the spatial distribution of the standing wave voltage for both cases, demonstrating the validity of the present design theory for the broadband booster circuit.
(a)

(b)


Fig. 7 Booster circuits designed by the electromagnetic wave simulator (HP Momentum), (a) Configuration of the booster circuit made of meander CPW transmission line, where spatial distribution of the standing wave voltage by EM simulator is shown, (b) Spatial distribution of the standing wave voltage expected from the transmission line model

## V. Conclusions

A new design theory for the broadband LN optical modulator for sub-carrier optical transmission is presented. Based on this theory we designed the practical device by EM simulator, and demonstrated the expected performances. Demonstration of the designed optical modulator using the flip chip bonding of the YBCO thin film on MgO substrate [6] is in progress.

## REFERENCES

[1] K. Yoshida, T. Uchida, S. Nishioka, Y. Kanda and S. Kojiro, "Design and performance of a velocity matched broadband optical modulator with superconducting electrodes," IEEE Trans. Appl. Supercond., vol 9, pp. 3905-3908, 1999.
[2] K. Yoshida, Y. Kanda, and S. Kohjiro, "A traveling-wave-type $\mathrm{LiNbO}_{3}$ optical modulator with superconducting electrodes," IEEE Trans. Microwave Theory Tech., vol 47, pp. 1201-1205, 1999.
[3] T. Sueta and M. Izutsu, "Integrated optic devices for microwave application," IEEE Trans. Microwave Theory Tech., vol. 38, pp. 477-482, 1990.
[4] N. Dagli, "Wide-bandwidth lasers and modulators for RF photonics," IEEE Trans. Microwave Theory Tech., vol 47, pp. 1151-1171, 1999.
[5] M. Izutsu, H. Murakami and T. Sueta, "Guided wave light modulator using a resonant coplanar electrode," IEICE Trans. Electron., vol. J71C, pp. 653-658, 1988.
[6] K. Yoshida, H. Morita, Y. Kanda, T. Uchiyama, H. Shimakage, and Z. Wang, "Optical modulator with superconducting resonant electrodes for sub-carrier optical transmission," Inst. Phys. Conf. Ser. No. 167, pp. 375-378, 2000.
[7] E. Rozan, C. Collado, A. Garcia, J. M. O'Callaghan, and R. Pous, "Design and fabrication of coplanar YBCO structure on Lithium Niobate," IEEE Trans. Appl. Supercond., vol 9, pp. 2866-2869, 1999.
[8] G. Matthaei, L. Young, and E. Jones, Microwave Filters, Impedance-Matching Networks, and Coupling Structures. New York: McGraw-Hill, 1964, pp. 427-440.
[9] K. Yoshida, K. Sashiyana, S. Nishioka, H. Shimakage, and Z. Wang, "Design and performance of miniaturized superconducting coplanar waveguide filters," IEEE Trans. Appl. Supercond., vol. 9, pp. 3905-3908, 1999.

