

# Superconducting Booster Circuits for Low Driving Voltage LiNbO<sub>3</sub> Optical Modulator

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**Abstract.** A general design theory of voltage and current booster circuits to excite large resonances in parallel and series resonators, which employ superconducting transmission lines, has been presented. Using the design theory, we designed and tested a broadband current amplifier for resonant type LiNbO<sub>3</sub> optical modulator using superconducting coplanar waveguide (CPW) transmission lines: the center frequency of 10 GHz, bandwidth of 500 MHz, and the current gain of 5.

## 1. Introduction

Applications of high-  $T_c$  superconducting transmission lines with low loss and low dispersion to microwave passive devices [1] have been studied extensively. We have been studying the applications of superconducting transmission lines to LiNbO<sub>3</sub> (LN) optical modulators of the traveling-wave-type[2],[3] as well as the resonant-type[4],[5]. In our previous paper [4], we proposed a novel design theory for current booster circuit for resonant type LN optical modulators for sub-carrier optical transmission (radio over fiber) [6], and experiments using YBCO films have been made [4],[5],[7].

In this paper, we present general design formulas for exciting both parallel resonator and series resonator, i.e., voltage and current booster circuits, and applied the theory to the design of 3-pole current booster circuit for resonant type LiNbO<sub>3</sub> optical modulator.

## 2. Design Theory

### 2.1 Design Formulas of Booster Circuits for Series and Parallel Resonators.

In Figs.1 (a) and (b), we show proposed equivalent circuits for current booster and voltage booster, respectively. The frequency characteristic of these circuits are designed to simulate the  $n$ -pole Chebyshev bandpass filters.

In Fig.1  $V_i$  is the complex amplitude of the incident wave from the signal source,  $Z_0=50\Omega$  is the system impedance,  $X$  and  $B$  are reactance and susceptance of the series resonator and parallel resonator to be excited, respectively. In the vicinity of the resonance frequency  $\omega_0$ , they can be represented as

$$X = x \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right), \quad (1)$$

$$B = b \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right), \quad (2)$$

where  $x = (\omega_0/2) \partial X / \partial \omega|_{\omega=\omega_0}$  and  $b = (\omega_0/2) \partial B / \partial \omega|_{\omega=\omega_0}$  represent reactance and susceptance slope parameters, respectively.

Parameters  $J_{i,i+1}$  and  $K_{i,i+1}$  ( $i=0,1,2,\dots,n$ ) represent inverters whose design values can be obtained by modifying the theory of n-pole Chebyshev lowpass filter with voltage source and current source, respectively [9] :

$$\begin{aligned} J_{01} &= \sqrt{\frac{wb_1}{Z_0 g_0 g_1}} \\ J_{i,i+1} &= w \sqrt{\frac{b_i b_{i+1}}{g_i g_{i+1}}} \quad (i=1,2,3,\dots,n-2) \\ J_{n-1,n} &= w \sqrt{\frac{b_{n-1} b'_n}{g_{n-1} g_n}} \\ J_{n,n+1} &= A_0 \sqrt{\frac{wb'_n}{Z_0 g_n}} \end{aligned} \quad (3)$$

with

$$b'_n = \frac{b_n}{1 - \frac{A_0^2 w x}{Z_0 g_n}} \quad (4)$$

and

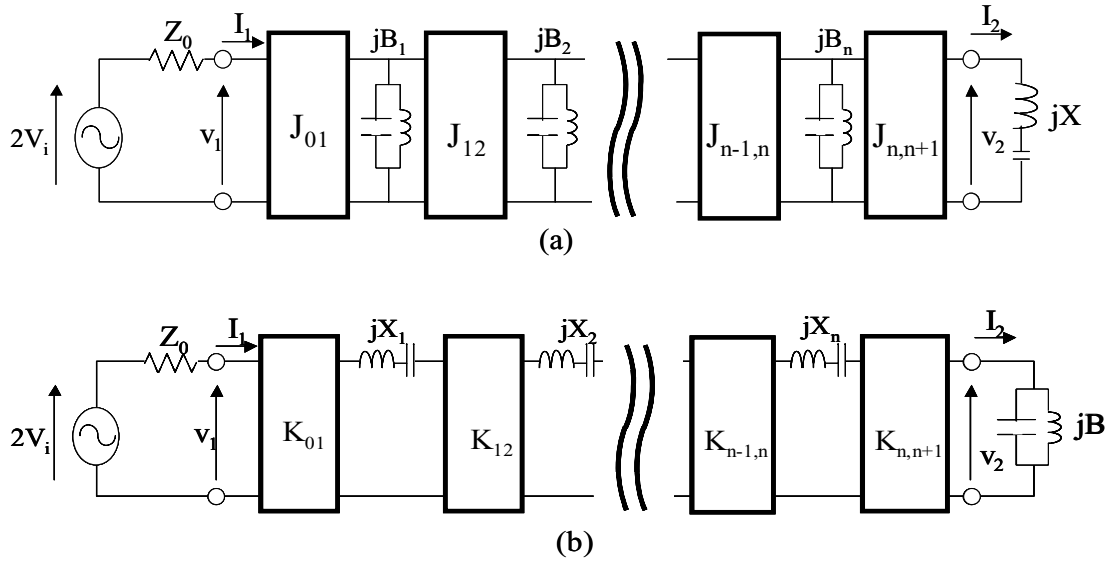
$$\begin{aligned} K_{01} &= \sqrt{\frac{w Z_0 x_1}{g_0 g_1}} \\ K_{i,i+1} &= w \sqrt{\frac{x_i x_{i+1}}{g_i g_{i+1}}} \quad (i=1,2,3,\dots,n-2) \\ K_{n-1,n} &= w \sqrt{\frac{x_{n-1} x'_n}{g_{n-1} g_n}} \\ K_{n,n+1} &= A_0 \sqrt{\frac{w Z_0 x'_n}{g_n}} \end{aligned} \quad (5)$$

with

$$x'_n = \frac{x_n}{1 - \frac{A_0^2 w b Z_0}{g_n}} \quad (6)$$

where  $g_i$  is an element value,  $w$  is the fractional bandwidth,  $b_i = (\omega_0/2) \partial B_i / \partial \omega|_{\omega=\omega_0}$  and  $x_i = (\omega_0/2) \partial X_i / \partial \omega|_{\omega=\omega_0}$  are susceptance and reactance slope parameters of the parallel and series resonant circuits,  $B_i$  and  $X_i$ , respectively,  $A_0$  is a parameter adjusting the

magnitude of the voltage and current amplification.



**Fig.1** Booster circuits for resonant circuits (a) current booster for series resonant circuit, (b) voltage booster for parallel resonant circuit.

In Fig.1, the voltage gain  $G_V$  and current gain  $G_I$  are given by

$$G_V = \frac{V_2}{V_i} = \frac{2(DZ_{in} - B)}{Z_0 + Z_{in}} \tag{7}$$

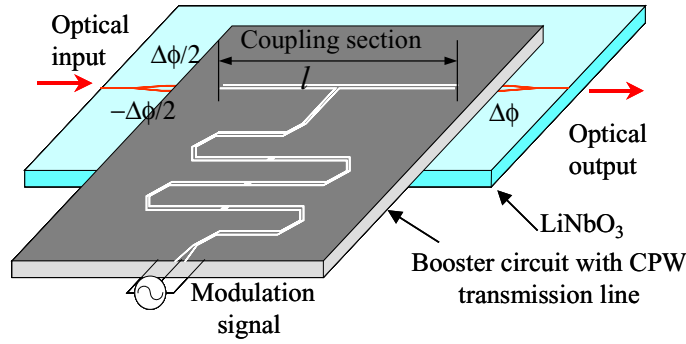
$$G_I = \frac{I_2}{V_i / Z_0} = \frac{2Z_0(A - CZ_{in})}{Z_0 + Z_{in}} \tag{8}$$

where parameters  $A, B, C, D$ , are the elements of the F matrix of the booster circuits,  $Z_{in} = V_i / I_1 = (AZ_L + B) / (CZ_L + D)$  is the input impedance,  $Z_L$  is the load impedance:  $Z_L = jX$  for current booster and  $Z_L = 1/jB$  for voltage booster.

It must be mentioned that  $b_n'$  and  $x_n'$  given by (4) and (6) are effective slope parameters of resonators, which incorporate the reactances of load resonators, and that in this design theory the voltage and current gains are designed so that  $|G_I| = |G_V| = A_0$  for passband and  $|G_I| = |G_V| = 0$  for offband.

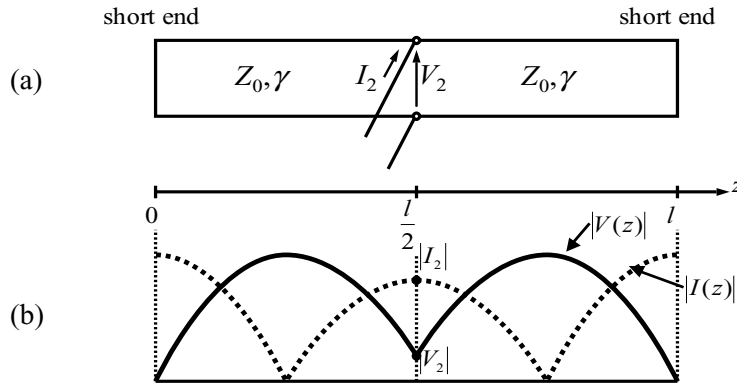
### 2.2 Design of 3-Pole Booster Circuit for Resonant Type Optical Modulator

In Fig.2 we show the schematic figure of the LN optical modulator with the booster circuit using superconducting coplanar waveguide (CPW) transmission lines. The modulation of lightwave is carried out throughout the Mach-Zehnder type optical waveguide via the standing-wave signal voltage occurring in the CPW resonator with a length  $l$ , which is short circuited at both ends.



**Fig.2.** Schematic of superconducting circuits on the Mach-Zehnder type LiNbO<sub>3</sub> optical modulator

As an example we designed 3-pole ( $n=3$ ) current booster circuit for exciting the series resonator as shown in Fig.1(a), which acts as the modulating electrode for the resonant type optical modulator[8]. In the present design we used the following parameters: center frequency  $f_0 = \omega_0/2\pi = 10$  GHz, bandwidth  $\Delta f = 500$  MHz corresponding to  $w = 5\%$ , the passband ripple of 0.1 dB. In this design the element values are:  $g_0 = 1$ ,  $g_1 = 0.5158$ ,  $g_2 = 1.0864$  and  $g_3 = 1.0895$ .



**Fig.3** Resonator for the resonant type optical modulator: (a) an equivalent transmission line circuit (b) spatial profiles of voltage and current at resonance

If we take the coordinate system as shown in Fig.3(a) assuming that the ends at  $z=0$  and  $z=l$  are short circuited, and that the resonator is connected to the booster circuit at  $z=l/2$ , the spatial profile of the standing wave voltage is given by

$$V(z) = V_2 \frac{\sinh[\gamma(l/2 - |z - l/2|)]}{\sinh(\gamma l/2)} \tag{9}$$

where  $\gamma = \alpha + j\beta$  is the propagation constant,  $\alpha$  is the attenuation constant,  $\beta = \omega\sqrt{LC}$  is the phase constant,  $\omega$  is the angular frequency,  $L$  and  $C$  are the inductance and capacitance of the CPW transmission line per unit length, respectively.

The input impedance of the resonator  $Z = V_2/I_2$  is also given by

$$Z = (Z'_0/2) \tanh(\gamma l/2) , \quad (10)$$

where  $Z'_0$  is the characteristic impedance of the coupling section . In the vicinity of the series resonance , i.e. ,  $\beta l = 2\pi$  at  $\omega = \omega_0$  , the expression for the reactance slope parameter  $x$  is given by

$$x = \frac{\omega_0}{2} \frac{\partial}{\partial \omega} \text{Im}[Z] \Big|_{\omega=\omega_0} = \frac{\pi}{4} Z'_0 . \quad (11)$$

In the presence of the standing-wave voltage, the induced total optical phase difference  $\Delta\phi(t)$  at the output end of the push-pull type optical modulator can be calculated as [5],[8]

$$\Delta\phi(t) = \frac{\pi}{V_\pi} \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) V_i(\omega) e^{j\omega t} d\omega , \quad (12)$$

with

$$F(\omega) = G_l(\omega) M(\omega) \quad (13)$$

$$\begin{aligned} M(\omega) &= \frac{1}{l} \int_0^l \frac{V(z)}{Z_0 I_2} e^{j\beta_0 z} dz \\ &= \frac{Z'_0}{2Z_0} \frac{(\gamma l/2) e^{j\beta_0 l/2} [\cosh(\gamma l/2) - \cosh(\beta_0 l/2)]}{[(\gamma l/2)^2 + (\beta_0 l/2)^2 \cosh(\gamma l/2)} \end{aligned} \quad (14)$$

where  $F$  is referred to as the normalized modulation depth,  $V_i(\omega)$  is the Fourier transform of the incident voltage waveform  $v_i(t)$  , i.e.,

$$V_i(\omega) = \int_{-\infty}^{\infty} v_i(t) e^{-j\omega t} dt , \quad (15)$$

and,  $V_\pi$  is the half-wavelength voltage,  $\beta_0 = (n_e/c)\omega$  ,  $n_e$  is the refractive index of light wave,  $c$  is the light velocity and  $M(\omega)$  is the term representing the magnitude of the coupling between signal and the optical fields, whose magnitude is of the order of unity.

### 3. Experiment

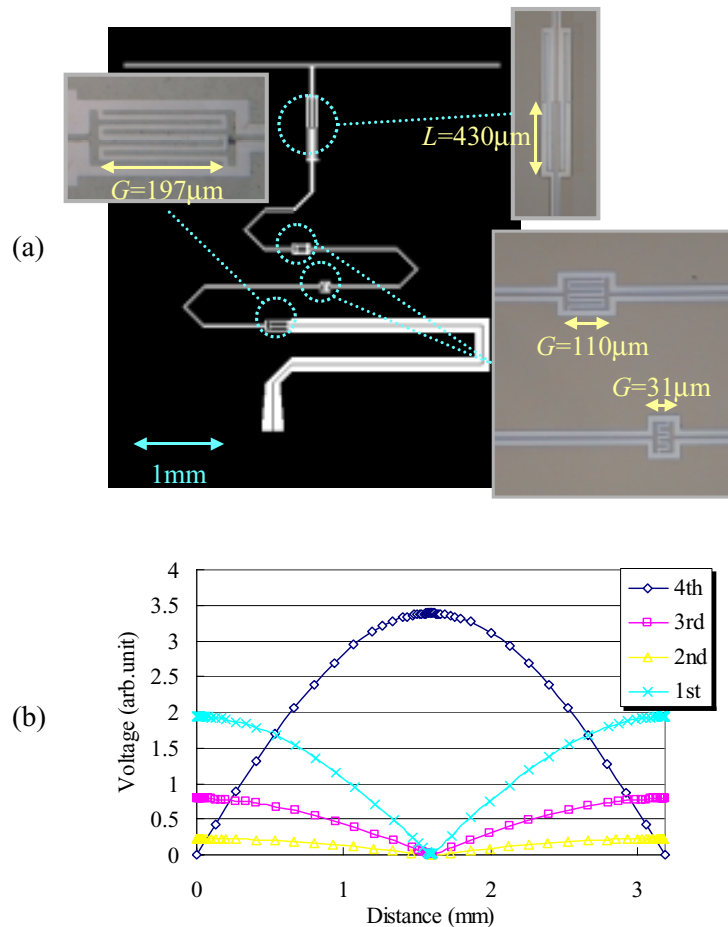
In order to realize a broadband booster circuit for exciting a large standing-wave voltage in the short circuited CPW as shown in Fig.3(a) and 3(b), we followed the procedures described in the design theory [5],[9]. In order to realize the parallel resonance circuits  $B_i$  and inverters  $J_{i,i+1}$  shown in Fig.1 (a), we use half-wavelength resonators and gaps in CPW transmission lines[10].

In Fig. 4(a), we show the configuration of the booster circuit made of meander CPW transmission lines. In Fig. 4(b) we show the spatial distribution of the amplitude of the standing wave voltage calculated by the transmission-line model, which is shown to be in a reasonable agreement with that obtained by the EM simulator.

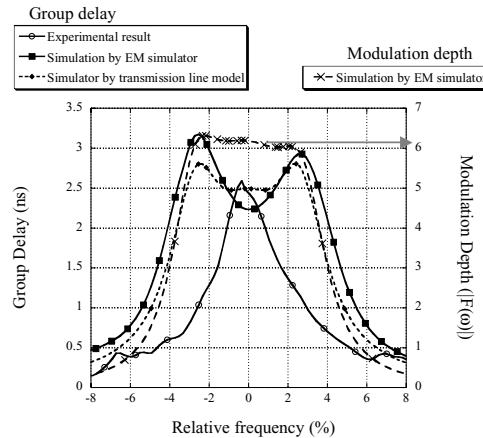
In Fig. 5, we show calculated frequency dependences of group delay

$\tau = -(\partial/\partial\omega)\arg S_{11}$ , where  $S_{11}$  is the scattering parameter at the input end  
 $S_{11} = (Z_{in} - Z_0)/(Z_{in} + Z_0)$ .

In order to confirm the proposed design theory we carried out the experiment. In Fig.4 (a) we show the pattern of the booster circuit with the meanderline 3-pole Chebyshev filter fabricated by a YBCO thin film on a MgO substrate with the dimension of 10[mm]x10[mm]. The YBCO electrode on MgO substrate was flip-chip bonded onto an optical waveguide in a LiNbO<sub>3</sub> substrate, and was cooled down by a refrigerator at T = 30K. In Fig. 5 we show the comparison of the observed group delay of  $S_{11}$  of the device and the simulation results. The observed data are similar to those expected from theory. The discrepancy between theory and experiment seems to be resulted from the precisions of the present preliminary experiment.



**Fig.4** Booster circuits designed by the electromagnetic wave simulator, (a) layout of the fabricated booster circuit, where insets show the layouts of J inverters. (b) spatial distribution of the standing-wave-voltage expected from the transmission line model



**Fig.5** Comparison of the frequency dependence of group delay between theory and experiment for  $w = 500\text{MHz} / 10\text{GHz} = 5\%$ . Modulation depth is also shown for reference.

#### 4. Conclusion

The design and performance of the superconducting booster circuit for a resonant type  $\text{LiNbO}_3$  optical modulator have been studied. Based on the design theory for the high gain booster, we designed the 3-pole current booster circuit using EM simulator, and tested the device performance.

#### Acknowledgments

This work was partly supported by a Grant-in-Aid for Scientific Research (B) from the Japan Society for the Promotion of Science (JSPS). This work was partly supported by the 21st Century COE Program 'Reconstruction of Social Infrastructure Related to Information Science and Electrical Engineering'

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